

History dependence and entropy How one can use entropy to solve Polya processes

R. Hanel, B. Corominas-Murtra, S. Thurner

Section of Science of Complex Systems, Medical University of Vienna, Austria

History-dependent stochastic processes are often non-ergodic and values of observables can no longer be predicted within the ensemble picture. The resulting mathematical difficulties pose severe limits to the analytical understanding of path-dependent processes. The statistics of such processes typically is not described by multinomial distributions. As a consequence, the multiplicities of similar paths of a process (or configurations of a system) is in general no longer given by the multinomial factor [1]. The maximum entropy principle based on Shannon entropy is tightly related to Bernoulli processes, systems with independent components, and to the ensemble picture; it loses its meaning for history-dependent processes, [2]. It is known that entropy, which takes the functional form of Shannon-entropy, is an equilibrium concept and that no unique generalization of entropy to systems operating out of equilibrium exist. However, this in fact implies that there may exist various useful generalizations of entropy to history-dependent processes, which all share the property that for Bernoulli processes they coincide with Shannon entropy, but take different functional forms if applied to other classes of processes. This is not entirely new. For instance, the information theoretic definition of entropy as information rates yields Shannon entropy for Bernoulli but the conditional entropy for Markov processes. Similarly, also the functional form of entropy used in maximum entropy principles to predict distribution functions, will depend on the process class of interest. Here we show how the ensemble picture can be avoided and the statistics of the underlying dynamical process is captured correctly in a functional that plays the role of a relative entropy, by construction. We demonstrate this for self-reinforcing Polya urn processes, which explicitly generalize multinomial statistics in a history dependent way. We demonstrate the adequacy of this constructive approach towards non-multinomial entropies by computing frequency and rank distributions of Polya urn processes [3].

We show how microscopic update rules of a path-dependent process allow us to explicitly construct a non-multinomial entropy functional, that, when maximized, predicts the time-dependent distribution function.

[1] R. Hanel, S. Thurner, B. Corominas-Murtra, PNAS **111**, 6905 (2014).

[2] B. Corominas-Murtra, R. Hanel, S. Thurner, PNAS **112**, 5348 (2015).

[3] R. Hanel, B. Corominas-Murtra, S. Thurner, New J. Phys. **19**, 033008 (2017).